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Nonlinear effects at high flux-flow electric fields

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Abstract
Ohm's law with the linear relation between resistive voltage and electric current is strictly valid only in the limit of infinitesimally small voltages. On the other hand, at finite electric voltages nonlinearities in the electric resistance can develop due to the energy picked up by the charge carriers in the electric field. This can lead to important effects both in the case of semiconductors and of superconductors, where the energy rise of the charge carriers or the quasiparticles can become relatively large. In this paper we limit our discussion to the flux-flow voltage in the mixed state of a type-II superconductor. At sufficiently low temperatures the energy dependence of the quasiparticle density of states and, hence, of the quasiparticle scattering rate can cause distinct nonlinear effects in the flux-flow resistance. The recent advances in thin-film sample preparation provided new opportunities for observing nonlinear effects of the latter kind.

1. Introduction

Nonlinear behavior of the electric resistance of materials can be observed frequently. Whereas Ohm's law with the linear relation between resistive voltage and electric current is strictly valid only in the limit of infinitesimally small applied electric fields, the energy picked up by the charge carriers in the electric field can lead to important nonlinear effects. Perhaps the most famous examples are encountered in the case of semiconductors, where the energy rise of the charge carriers can lead to the occupation of another band with a larger effective mass (Gunn effect), or where it results in impact ionization of the doping impurities and avalanche breakdown (current filaments). Another type of important examples is found in the case of flux-flow resistance developed in the mixed state of type-II superconductors. In both kinds of examples it is the relatively low concentration of the relevant electric charge carriers which allows the application of a relatively large electric field and, hence, an appreciable amount of energy to be picked up by them in the electric field without resulting in an intolerably large electric current density and Joule heating. On the other hand, in metals, due to their large concentration of charge carriers, sufficiently large electric fields for promoting nonlinear effects in the electric resistance cannot be generated without excessive Joule heating.

In this paper we restrict our discussion only to purely electronic phenomena without including Joule heating effects. Furthermore, we only deal with the flux-flow resistance in the mixed state of a type-II superconductor. In order to eliminate or minimize Joule heating effects, efficient cooling of the samples is necessary. This can be accomplished using the geometry of extremely thin films having a large surface/volume ratio. The recent advances in thin-film technology, with the film thickness approaching the nanometer scale, provides a reliable opportunity for such experiments. Finally, we do not deal with flux pinning.

2. Three energy scales

A brief discussion of the electronic structure of a single vortex and of the vortex lattice can be found in [1]. In the case of the quasiparticles in the vortex cores we must distinguish three important energy scales: the superconducting energy gap Δ, the level spacing Δ^2/ε_F of the Andreev bound states and the energy smearing $\delta\varepsilon = \hbar/\tau$ due to the mean electronic scattering time τ. (ε_F = Fermi energy; $\hbar$ = Planck's constant divided by 2π). Correspondingly, we have the following three limits: the 'dirty' limit for $\Delta \ll \delta\varepsilon$, the moderately clean limit for $\Delta^2/\varepsilon_F \ll \delta\varepsilon \ll \Delta$ and the 'superclean' limit for $\delta\varepsilon \ll \Delta^2/\varepsilon_F$.

In the case of the dirty limit any energy dependence of the density of states and of the scattering properties of the quasiparticles is smeared out due to the large scattering rate. The vortex can be described simply in terms of a cylinder

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of normal phase with its radius given by the superconducting coherence length ξ. (So far, we assume s-wave symmetry of the pair wavefunction.) In this case the flux-flow resistivity ρ_{fl} is well accounted for by the normal resistivity ρ_{n} of the volume fraction occupied by these cylinders, as expressed by the Bardeen–Stephen model [2]:

$$\rho_{\mathrm{fl}} = (B/B_{\mathrm{C2}})\rho_{\mathrm{n}} \tag{1}$$

(B = magnetic flux density, B_{C2} = upper critical value of B). The Bardeen–Stephen model is generally satisfied by classical superconductors. Deviations from the Bardeen–Stephen model may appear in highly pure superconductors in the limit of very low temperatures, where the dirty limit is not valid any more and the electronic quantum structure of the vortices can have an effect.

In the case of the moderately clean limit, also referred to as the quasiclassical limit, the energy dependence of the electronic structure of the vortices cannot be ignored any more. However, the discrete energy spectrum can be treated as a quasicontinuum. Finally, the full quantum structure of the energy spectrum must be taken into account only in the case of the superclean limit (Bogoliubov–de Gennes equations). References can be found in [1].

In contrast to the classical superconductors, the cuprate (high-temperature) superconductors show predominantly d-wave symmetry of the pair wavefunction with the energy gap vanishing at the node lines. Furthermore, in the cuprates the coherence length ξ is extremely small, being up to 100 times smaller than in the classical superconductors and reaching values of only 1–2 nm. Due to this fact, the level spacing $\Delta^2/\varepsilon_{\mathrm{F}} \sim 1/\xi^2$ of the Andreev bound states is up to 10 000 times larger than in the classical superconductors, and electronic quantum effects may become important.

3. Nonlinear flux-flow resistance

Nonlinear effects in the flux-flow resistance of purely electronic origin have been discussed for some time. In 1975 Larkin and Ovchinnikov have shown that the energy increase of the quasiparticles in the vortex core due to the electric field generated by vortex motion results in a shrinking of the vortex core, a corresponding reduction of the damping coefficient of vortex motion and an instability of the flux-flow resistance [3]. This effect is expected in the high-temperature limit ($T \approx T_{\mathrm{c}}$) and has been observed experimentally both in classical and high-temperature superconductors. In the low-temperature limit ($T \ll T_{\mathrm{c}}$) Larkin and Ovchinnikov predicted a logarithmic singularity of the voltage–current characteristic associated with the thermal population of the bound states in the vortex core and a current-dependent effective temperature T^* of the quasiparticles [4]. A brief discussion of these effects and further references can be found in [1] and [5].

Apparently, so far a theoretical discussion of possible nonlinear effects in the flux-flow resistance, taking into account the quantum structure of the energy spectrum of the quasiparticles in the mixed state, has not been reported. The theoretical treatments reported up to now were all restricted to stationary vortices or to the linear regime of vortex motion. However, at sufficiently low temperatures interesting phenomena can be expected. We quote from a paper by Rainer *et al* [6]: having indicated that the physics of vortex cores in clean superconductors ($\xi \ll \ell$) is very different from the physics of the vortex core in a dirty superconductor ($\ell \ll \xi$), they point out '$\cdots$*We expect more spectacular effects in the dynamic properties*$\cdots$' leading to a situation *'which will produce a rich spectrum of largely unexplored dynamical phenomena.'*

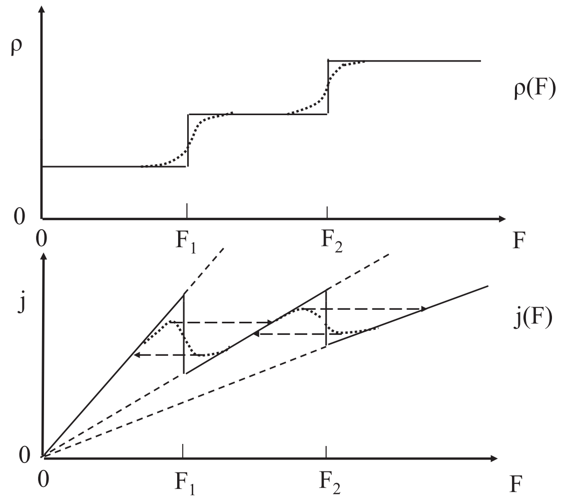

Figure 1. Electric resistivity ρ plotted versus the electric field F showing two steps at the field values F_1 and F_2, respectively (top), and the resulting electric current density j plotted versus the field F (bottom).

We illustrate this regime by the following simple example. We assume a situation where the resistivity $\rho(F)$ shows abrupt steps at the electric field values F_1 and F_2 (see figure 1). At these values of the field the current density j switches to a lower value given by the higher resistivity. If the steps in the resistivity are smeared, the current density j also more gradually shifts to the lower value (dotted lines). For current-biased operation, at the onset (offset) of the negative differential conductivity the current density switches to the next higher (lower) stable branch of the characteristics, showing hysteretic behavior.

A key element in this discussion is the energy shift of the Fermi surface along the direction of the electric field F by the amount [7]

$$\delta\varepsilon_{\mathrm{k}} = eFv_{\mathrm{k}}\tau. \tag{2}$$

The subscript k indicates the point on the Fermi surface in **k**-space. v_{k} is the quasiparticle velocity and e the elementary charge. We assume that the thermal energy smearing $k_{\mathrm{B}}T$ is small compared to the energy shifts $\delta\varepsilon_{\mathrm{k}}$ effected by the fields F_1 and F_2 shown in figure 1.

As we see from (2), the energy shift $\delta\varepsilon_{\mathrm{k}}$ of the quasiparticles is proportional to the scattering time τ. If the time τ depends on the quasiparticle energy, the resistivity becomes dependent on the electric field, resulting in a situation such as shown in figure 1 and a nonlinearity in the resistivity. If electron–electron scattering is dominant, as is the case in the cuprate superconductors at very low temperatures, we have

$1/\tau \sim N^2(\varepsilon)$, where $N(\varepsilon)$ is the density of states providing the available phase space for scattering. From the Drude expression for the resistivity one finds

$$\rho \sim \frac{1}{N}\frac{1}{\tau} \sim \frac{N^2}{N} \sim N. \tag{3}$$

Together with (2) we see that an energy dependence $N(\varepsilon)$ can result in a field dependence $\rho(F)$.

In the case of s-wave symmetry of the pair wavefunction, the electronic quantum structure of the vortex lattice results from the Andreev bound states of an isolated vortex and the level spacing Δ^2/ε_F. In the case of d-wave symmetry of the pair wavefunction (as for the cuprates), the existence of the node directions with zero energy gap has a strong effect. However, as shown by Kopnin and Volovik [8, 9], in this case there also exists an *average minigap* of the same order Δ_0^2/ε_F as in the s-wave case. Furthermore, there is a characteristic resonant energy $(\Delta_0^2\hbar\omega_c/\varepsilon_F)^{1/2}$, where ω_c is the cyclotron frequency. So in the clean limit the density of states can be expected in general to show an energy dependence $N(\varepsilon)$, which can lead to nonlinear behavior of the flux-flow resistance. A detailed discussion of this argument for explaining the experimentally observed instabilities in the resistivity of the electron-doped superconductor $Nd_{2-X}Ce_XCuO_Y$ can be found in [10]. We note that here the energy dependence of the density of states in the case of the quasiparticle scattering rate plays a role somewhat similar to the temperature dependence of the specific heat in the case of the Schottky anomaly.

4. Resistivity of the vortex core

Having discussed the nonlinear effects in the flux-flow resistance, next we turn to the procedure for extracting the resistivity of the vortex core from the measured current–voltage characteristic. This turns out to be interesting, if one compares samples belonging to the dirty and the clean limit, respectively. Again, the theoretical background has been given by Larkin and Ovchinnikov [3, 11]. We illustrate this point by means of a brief discussion of previous experiments performed with superconducting amorphous Mo_3Si films (dirty limit) and epitaxial c-axis-oriented $YBa_2Cu_3O_{7-\delta}$ films (clean limit) [12]. In order to eliminate or minimize the influence of flux pinning, these measurements were carried out at high vortex velocities. For details we refer to [12].

Extending the theory of the flux-flow resistivity to the regime of high vortex velocities, in the dirty limit Larkin and Ovchinnikov obtained for the relation between electric current density j and electric field F

$$j = \frac{1}{\rho}F\left[\frac{1}{1+(F/F^*)^2} + c\left(1-\frac{T}{T_C}\right)^{1/2}\right]. \tag{4}$$

Here c is of the order of unity, ρ is a reduced flux-flow resistivity and F^* is a critical electric field value. If the second term in the bracket on the rhs would be absent, with increasing electric field the current density would pass through a maximum at $F = F^*$, reaching a regime with negative differential conductivity. Hence, at $F = F^*$ an electronic instability is reached, and in the case of current bias the sample switches into a state with higher resistivity. At the critical electric field F^* a distinct kink appears in the I–V characteristic, which can be detected easily by experiment. At the instability point $F = F^*$, according to (4) the current density is

$$j^* = \frac{1}{\rho}F^*\left[\frac{1}{2} + c\left(1-\frac{T}{T_C}\right)^{1/2}\right]. \tag{5}$$

The resistivity ρ is given by

$$\rho = \rho_{core}\left[1 + \frac{1}{(1-T/T_C)^{1/2}}\frac{B_{C2}}{B}f(B/B_{C2})\right]^{-1}. \tag{6}$$

Here ρ_{core} denotes the core resistivity. In the dirty limit we have $\rho_{core} = \rho_n$. For the function $f(B/B_{C2})$ in (6) Larkin and Ovchinnikov obtained an expression which is also given in equation (5) of [12]. Hence, if j^* and F^* are measured, one can find the core resistivity using (5) and (6). Due to the fact that the data are obtained at high vortex velocities, the results are much less affected by flux pinning than measurements carried out in the limit of small vortex velocities.

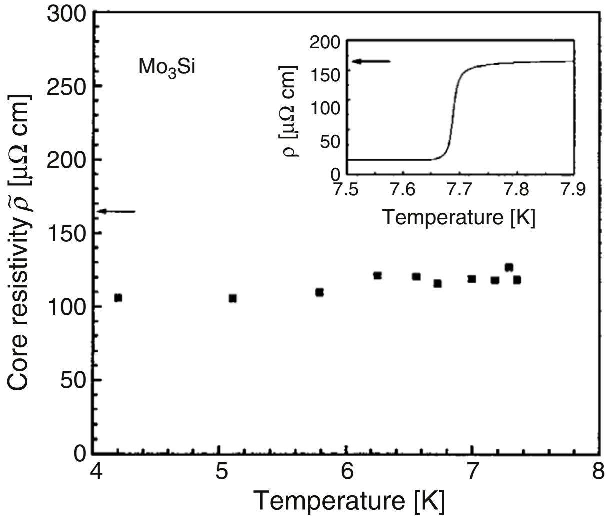

Figure 2. Core resistivity ρ_{core} versus temperature for amorphous Mo_3Si. The inset shows the resistive transition. The normal-state value is marked by the arrow. Reproduced with permission from [12]. Copyright 1996 by IOP Publishing.

Results obtained from experiments performed with superconducting amorphous Mo_3Si films and taken from [12] are shown in figure 2. The core resistivity ρ_{core} calculated from (5) and (6) is plotted versus temperature. The inset indicates the resistive transition. The normal-state resistivity is marked by the arrow. The core resistivity is seen to be nearly temperature-independent. It is slightly smaller than the normal-state value.

Next we turn to the case of the clean limit. In this case Larkin and Ovchinnikov obtained the relation

$$\rho = \rho_{core}\left[1 + \frac{0.46}{\pi}\frac{B_{C2}}{B}\frac{\Delta(T)}{k_BT}\right]^{-1}. \tag{7}$$

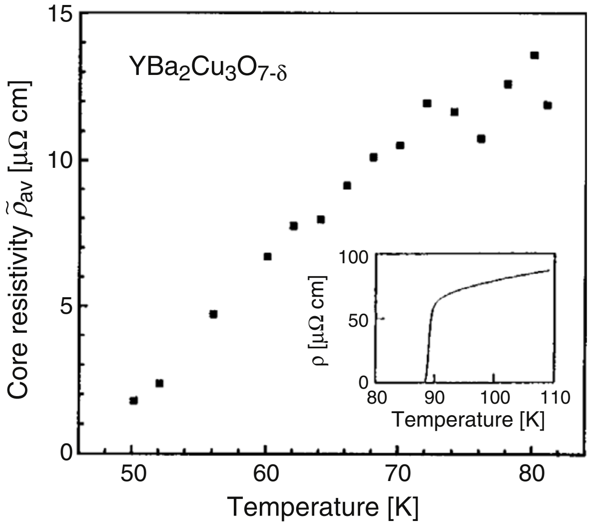

Figure 3. Core resistivity ρ_{core} versus temperature for epitaxial c-axis-oriented $YBa_2Cu_3O_{7-\delta}$. The inset shows the resistive transition. Reproduced with permission from [12]. Copyright 1996, by IOP Publishing.

Here k_B is Boltzmann's constant. The nonlinearity of the I–V characteristic is taken into account by the relation

$$j = \frac{1}{\rho}F\left[\frac{1}{1+(F/F^*)^2}\right]. \quad (8)$$

In figure 3 we show results obtained from experiments performed with epitaxial c-axis-oriented $YBa_2Cu_3O_{7-\delta}$ films and taken again from [12]. The core resistivity calculated from (7) and (8) is plotted versus temperature. The inset shows the resistive transition. For simplicity and ignoring the d-wave symmetry of the pair wavefunction, the temperature-dependent function $\Delta(T)$ was taken from the BCS theory using $\Delta(0) = 3.5k_BT_c$. The core resistivity is seen to decrease strongly with decreasing temperature, near 50 K approaching values almost two orders of magnitude smaller than the normal-state value.

Apparently, in the clean limit the physics of the core resistivity is distinctly different from that in the dirty limit, as emphasized in the quotation from [6] presented above. In the dirty limit, the vortex core can be well described in terms of a cylinder of normal phase with the radius ξ (Bardeen–Stephen model). In the clean limit, the quasiparticle scattering rate in the vortex core becomes strongly temperature-dependent and the electronic quantum structure of the core is expected to be more and more important at low temperatures.

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